Announcements

1. A1 due Thu 4 Sep 8 pm
2. Mid-Sem Feedback (see Ed)
   Due on Sep 10.
3. Appendix (Set notation)

Today

1. Finish Intro to FA (4.1.5 - 4.1.7)
2. Operations on Languages (4.2)
3. Non-determinism (4.3)

Def (Finite Automata)

① Alphabet Σ (e.g. $\{0,1\}$, {"res", "inc"}.) $\{W, A, S, D\}$.

② Set of states Q

$$2^{\{0,1\}} = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}.$$

③ Initial state q_0

④ State transition function $\delta : Q \times \Sigma \to Q$

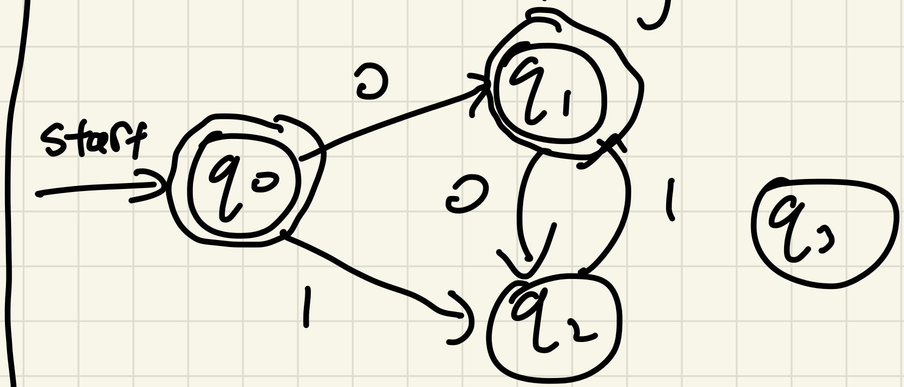
δ(curr_state, next_char) = next_state.

⑤ Accept states F (subset of Q).

State Transition Table

	0	1
q_0	q_1	q_2
(accept) q_1	q_2	
q_2		q_1
q_3		

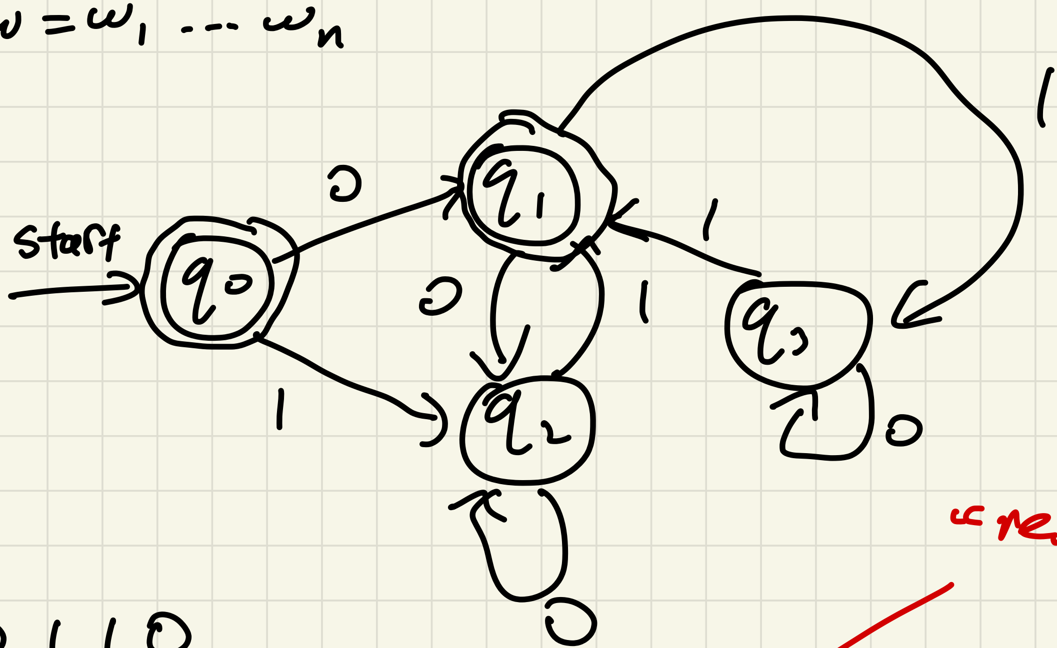
State Diagram



Computation Path / Accept / Reject

String w, FA M

$w = w_1 \cdots w_n$



0110

$q_0 \longrightarrow q_1 \longrightarrow q_3 \longrightarrow q_2$

"resulting state"

Def M accepts w if resulting state is in F

$L(M)$ = set of strings it accepts

Regular Languages

Def

FA M recognizes L iff $L = L(M)$

Def (Regular Languages)

A language L is regular iff $\exists$ FA M that recognizes L, i.e. $L = L(M)$.

Key features of FA

1. Memoryless

   States = memory and constant # states

2. Input stream

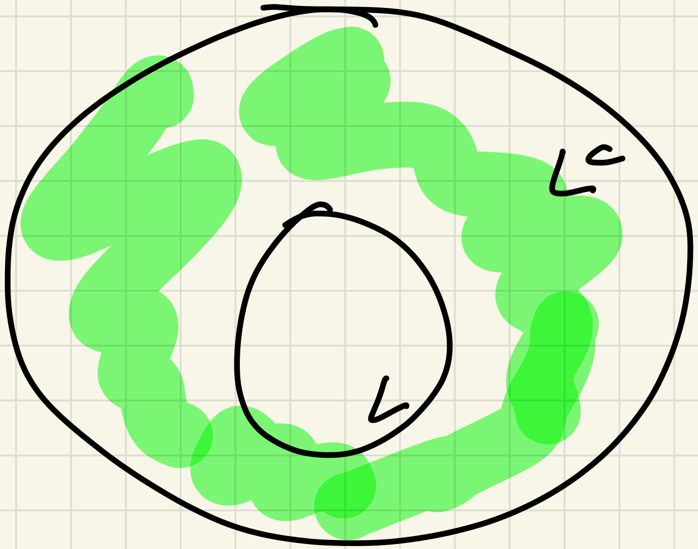
   Don't know when character is last

Demo on Ed

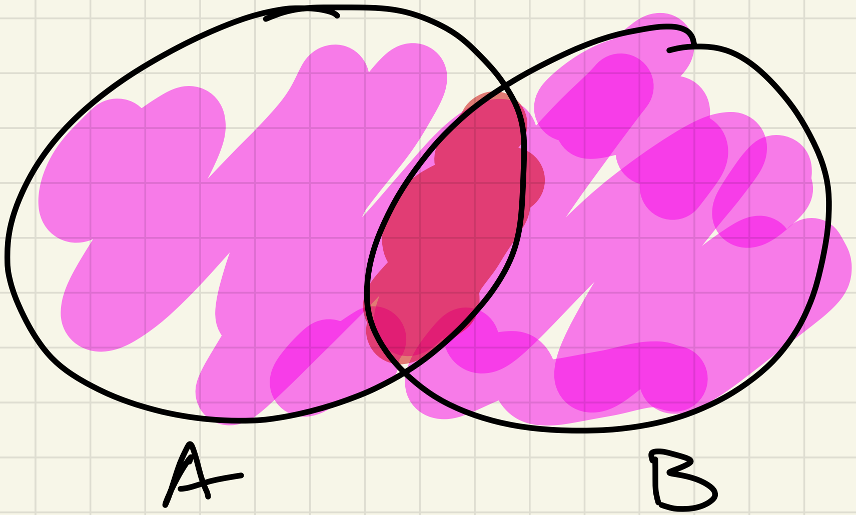
Visualise internal state on pythontutor.com.

Operations on Languages

1. Complement
2. Intersection
3. Union



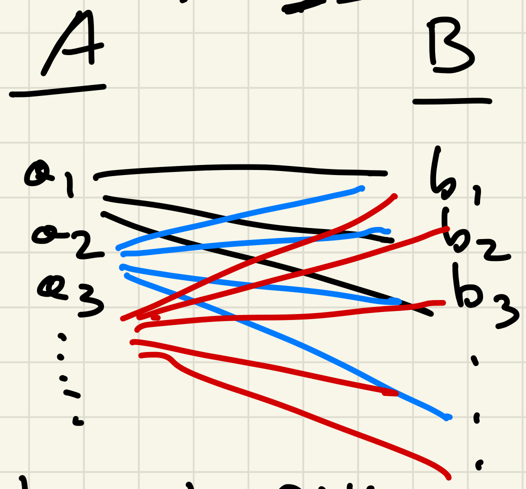
$\Sigma^* =$ set of strings w/ alphabet Σ



④ Concatenation of A and B

$$(A \circ B) \circ C = A \circ (B \circ C)$$

$$\{a_1\} \circ B = \{a_1 b_1, a_1 b_2, a_1 b_3, \ldots$$



e.g. $A = \{0, 00, 000, 0000, \ldots\}$

$B = \{1, 11, 111, 1111, \ldots\}$

all-ones. $\quad A \circ B = \{01, 011, 0111$

$$A \circ B = (\{a_1\} \circ B) \cup (\{a_2\} \circ B) \cup \ldots$$

⑤ Kleene star

(aka Kleene closure)

Language L

$$L^* = L^0 \cup L^1 \cup \cdots \cup L^k \cup \ldots$$
$$= \{L^k : k \geq 0\}.$$

Define $L^0 = \{\varepsilon\}$, $L^1 = L$, $L^2 = L \circ L$

$$L^k = L \circ \cdots \circ L = L^{k-1} \circ L$$

$$L^k = \underbrace{L \circ \cdots \circ L}_{k \text{ times}}.$$

$$L^* = \{\varepsilon\} \cup L \cup \underbrace{(L \circ L)}_{L^2} \cup \underbrace{(L \circ L \circ L)}_{L^3} \cup \cdots$$

$$L^* = \{\varepsilon\} \cup \{z : \exists\, k \geq 1 \text{ and } x_1, \ldots, x_k \in L \text{ and } z = x_1 x_2 \cdots x_k\}.$$

Closure Properties

① L regular $\Rightarrow L^c$ and L^* are regular

② A, B regular $\Rightarrow A \cup B$, $A \cap B$, $A \circ B$ regular.

Concatenation, union, Kleene star called "regular operations"

Nondeterministic FA (NFA)

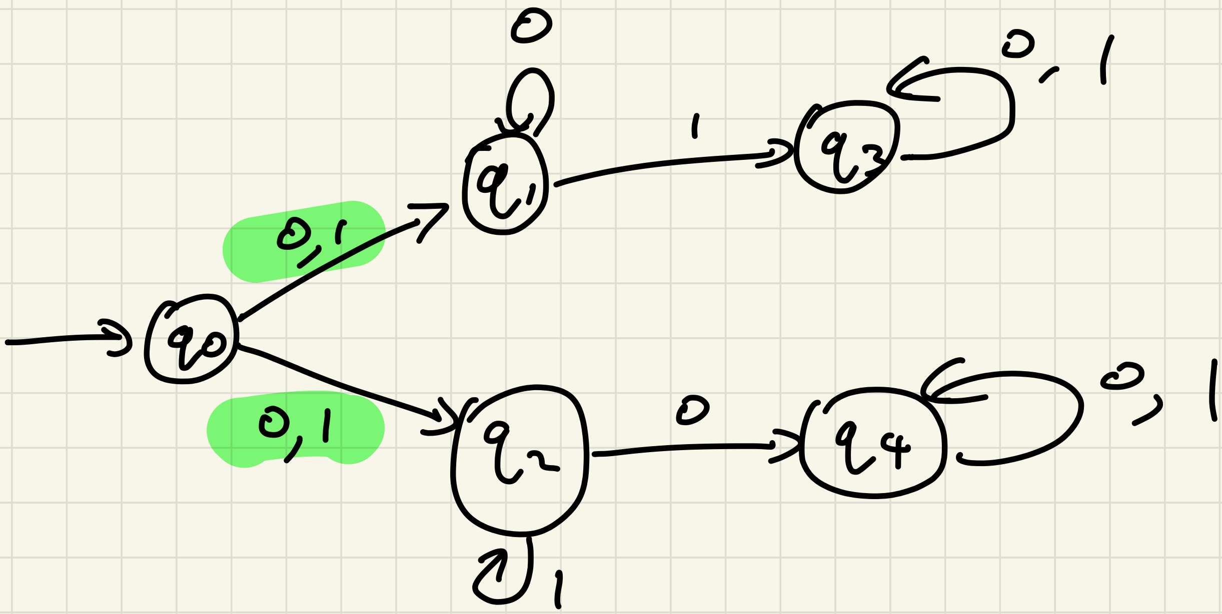
Def (NFA w/o ε-transitions)

1. Alphabet Σ
2. Set of states Q
3. Initial state q_0
4. State transition function $\delta : Q \times \Sigma \to 2^Q$

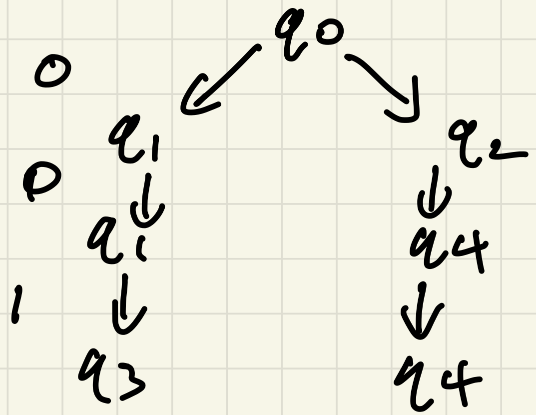
   $\delta(\text{curr\_state}, \text{next\_char}) = \text{next\_states}$
5. Accept states F (subset of Q)

④ State transition function $\delta : Q \times \Sigma \to 2^{Q}$

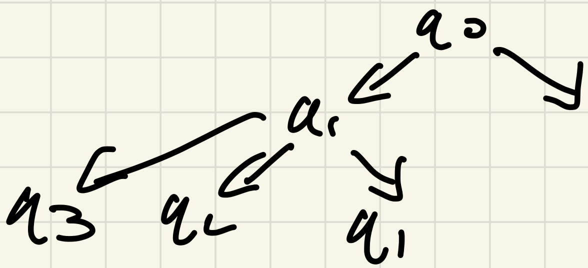
$\delta(\text{curr\_state}, \text{next\_char}) = \text{next\_states}$



0 0 1



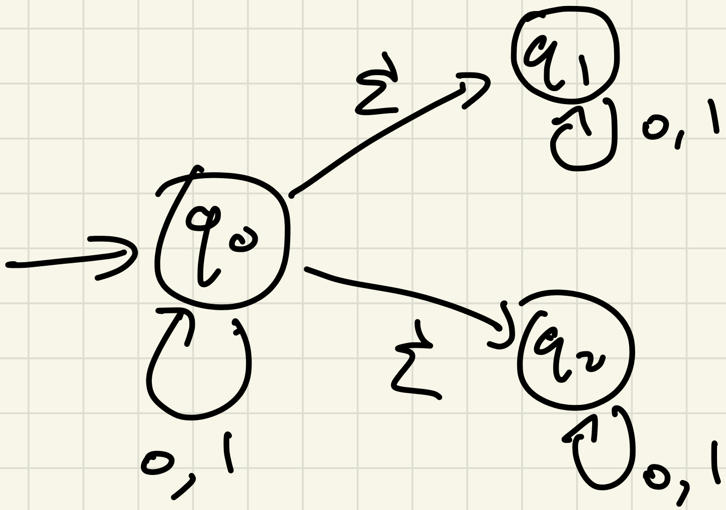
$\delta(q_0, 1) = \{q_1, q_2\}$.



<u>Def</u> NFA N accepts w iff

there exists an accepting computation path.

ε transitions



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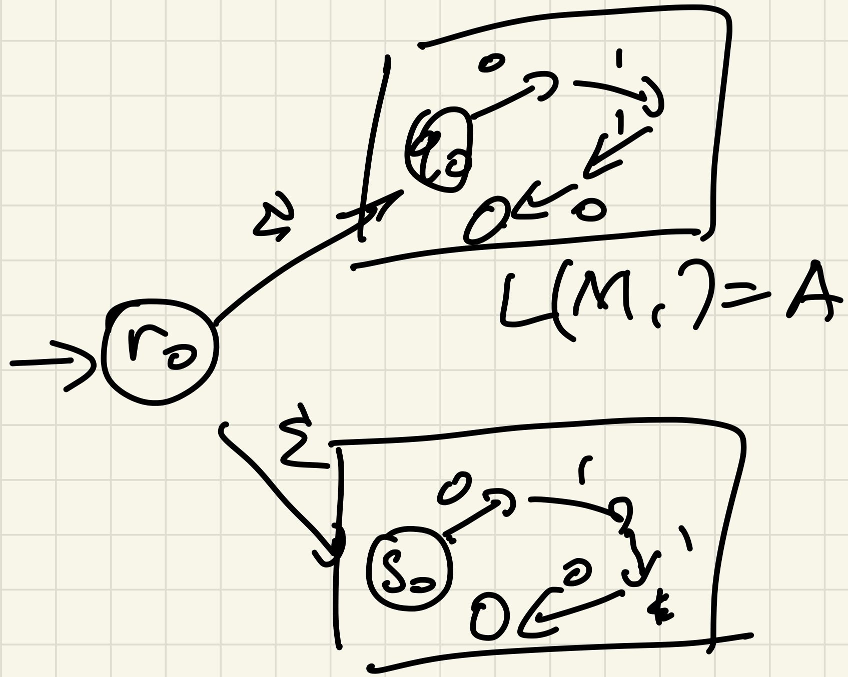
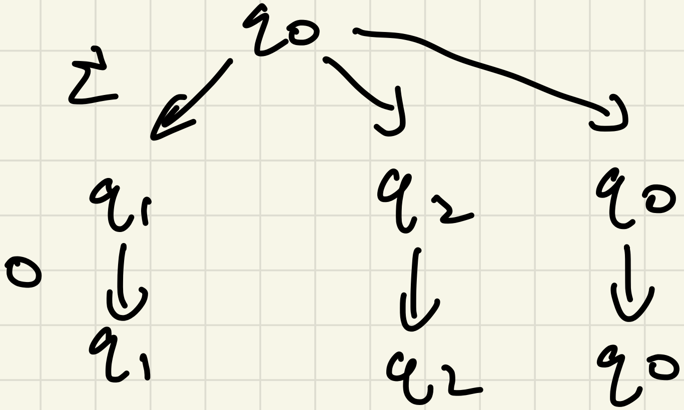


$L(M_2) = B$

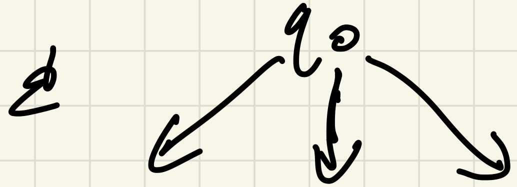
Input 01



$\underline{0110} = \varepsilon 0 1 \varepsilon \varepsilon 1 0$

$1\varepsilon = 1$

$0\varepsilon = 0$



$$q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{\varepsilon} q_2$$

$$q_0 \xrightarrow{0,1} q_1 \quad = \quad \begin{tikzcd} q_0 \arrow[r, bend left, "0"] \arrow[r, bend right, "1"'] & q_1 \end{tikzcd}$$